

Some remarks about level sets of Cesaro averages of binary digits

G. CARDONE, A. CORBO ESPOSITO, L. FAELLA

Abstract

The problem of averaging the binary digits of numbers in $[0, 1]$ is considered. It is well-known that Lebesgue a.e. in $[0, 1]$ the usual Cesaro average is equal to $\frac{1}{2}$ and that the Hausdorff dimension of the set where the Cesaro average is equal to α is given by an entropy function $d(\alpha)$. we prove that if $\alpha \neq \frac{1}{2}$ then the Hausdorff measure $\mathfrak{H}^{d(\alpha)}$ of such aset is infinite. We more explicitly construct an infinite matrix T (in a class \mathfrak{M} of Toeplitz matrices regular with respect to Cesaro averages) such that the hausdorff dimension of the set of the points not having Cesaro average and where the T -generalized average is α is still given by $d(\alpha)$.